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ANTIGLARE SUPPORTS AND CONTRAST AMPLIFYING SUPPORTS FOR POLARISED LIGHT REFLECTION

The present invention concerns object-carrying supports intended for improving the measurements or the reflection observation of thin films or of very small objects under optical microscope or under any other optical imaging instrument: visor, glasses, macroscope, magnifying glass, binocular

5   magnifying glass, camera, photo camera, near-filed microscope, endoscope, cofocal microscope, near field optical microscope (SNOM), biochip readers, magneto-optical reader, confocal microscope. These supports are in particular intended for use in visualisation and measuring techniques in reflection differential interferential contrast (DIC). They are also intended for use in all

10   observation and measuring techniques in polarised light reflection.

These supports are defined either by a set of characteristics which may concern their nature (example: a dielectric layer on a silicon substrate) or their function (example: contrast amplifying support for the observations between crossed polarisers), and which, if they are common to a certain number of

15   these supports, class it in families of supports, or by a set of adjustable parameters which enable to recognise individually the members of a given family.

The object of the present invention is therefore to offer a support, exhibiting in relation to the thickness e_1 and to the index n_1 of the layer formed

20   on a substrate of complex refraction index n_2, in a surrounding medium of index n_0, either a contrast amplifying function for a polarised light observation, or a antiglare function for the polarised light.

To this effect, the invention concerns a support intended for observing between crossed polarisers an object placed on the support or in the vicinity

25   thereof in a medium of index n_0 with spatially incoherent incident convergent illumination under an angle θ_0 at a wavelength λ, including

- a substrate of complex refraction index n_2,
- a layer of complex refraction index n_1 and of thickness e_1.

The value of the thickness e_1 of the layer is within 2 % so that:

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$$\frac{d^2}{de_1^2} \ln|\sigma|^2 = 0$$

with

$$\sigma = \frac{\sigma_{01} + \sigma_{12}(1 + \pi_{01})e^{(-2j\beta_1)} + \sigma_{01}\pi_{12}e^{(-4j\beta_1)}}{(1 + r_{01(p)} + r_{12(p)}e^{(-2j\beta_1)})(1 + r_{01(s)}r_{12(s)}e^{(-2j\beta_1)})}$$

a formula wherein σ_{ij} and π_{ij} represent respectively the sum and the product of the Fresnel coefficients of the different interfaces [(i,j)=(0,1) or (1,2)]:

$$r_{ij(p)} = \frac{n_j \cos\theta_i - n_i \cos\theta_j}{n_j \cos\theta_i + n_i \cos\theta_j}$$

and

$$r_{ij(s)} = \frac{n_i \cos\theta_i - n_j \cos\theta_j}{n_i \cos\theta_i + n_j \cos\theta_j}$$

and wherein $\beta_1 = \dfrac{2\pi n_1 e_1 \cos\theta_1}{\lambda}$,

with $\cos\theta_1 = \sqrt{1 - \left(\dfrac{n_0}{n_1}\right)^2 \sin^2\theta_0}$.

The invention also concerns a support intended for observing between crossed polarisers an object placed on the support or in the vicinity thereof in a medium of index n_0 with incident convergent incoherent illumination under an angle θ_0 at a wavelength λ, including

- a substrate of complex refraction index n_2,
- a layer of complex refraction index n_1 and of thickness e_1.

The value of the thickness e_1 of the layer is within 2 % so that:

$$\frac{d}{de_1}\left|\sigma^2\right| = 0$$

with

$$\sigma = \frac{\sigma_{01} + \sigma_{12}(1 + \pi_{01})e^{(-2j\beta_1)} + \sigma_{01}\pi_{12}e^{(-4j\beta_1)}}{(1 + r_{01(p)} + r_{12(p)}e^{(-2j\beta_1)})(1 + r_{01(s)}r_{12(s)}e^{(-2j\beta_1)})}$$

a formula wherein σ_{ij} and π_{ij} represent respectively the sum and the product of the Fresnel coefficients of the different interfaces [(i,j)=(0,1) or (1,2)]:

$$r_{ij(p)} = \frac{n_j \cos\theta_i - n_i \cos\theta_j}{n_j \cos\theta_i + n_i \cos\theta_j}$$

and

$$r_{ij(s)} = \frac{n_i \cos\theta_i - n_j \cos\theta_j}{n_i \cos\theta_i + n_j \cos\theta_j}$$

and wherein $\beta_1 = \dfrac{2\pi n_1 e_1 \cos\theta_1}{\lambda}$,

5   with $\cos\theta_1 = \sqrt{1 - \left(\dfrac{n_0}{n_1}\right)^2 \sin^2\theta_0}$.

The invention also concerns a support intended for optimising the useful extinction coefficient of a polarising microscope for observing an object placed on the support or above the support in a medium of index n_0 with

10   incident convergent incoherent illumination under an angle θ_0 at a wavelength λ, including

- a substrate of complex refraction index n_2,
- a layer of complex refraction index n_1 and of thickness e_1.

The value of the thickness e_1 of the layer is within 2 % so that:

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$$\frac{d}{de_1}\left(\frac{|\sigma|^2}{R_{NP}}\right) = 0$$

with

$$R_{NP} = \frac{1}{4}\left|r_p + r_s\right|^2 + \frac{1}{4}\left|r_p - r_s\right|^2$$

and

20    $$r_p = \frac{r_{01(p)} + r_{12(p)}e^{(-2j\beta_1)}}{1 + r_{01(p)}r_{12(p)}e^{(-2j\beta_1)}} \quad \text{and } r_s = \frac{r_{01(s)} + r_{12(s)}e^{(-2j\beta_1)}}{1 + r_{01(s)}r_{12(s)}e^{(-2j\beta_1)}}$$

and

$$\sigma = r_p + r_s = \frac{\sigma_{01} + \sigma_{12}(1 + \pi_{01})e^{(-2j\beta_1)} + \sigma_{01}\pi_{12}e^{(-4j\beta_1)}}{(1 + r_{01(p)} + r_{12(p)}e^{(-2j\beta_1)})(1 + r_{01(s)}r_{12(s)}e^{(-2j\beta_1)})}$$

a formula wherein σ_{ij} and π_{ij} represent respectively the sum and the product of

25   the Fresnel coefficients of the different interfaces [(i,j)=(0,1) or (1,2)]:

$$r_{ij(p)} = \frac{n_j \cos\theta_i - n_i \cos\theta_j}{n_j \cos\theta_i + n_i \cos\theta_j}$$

and

$$r_{ij(s)} = \frac{n_i \cos\theta_i - n_j \cos\theta_j}{n_i \cos\theta_i + n_j \cos\theta_j}$$

and wherein $\beta_1 = \dfrac{2\pi n_1 e_1 \cos\theta_1}{\lambda}$, with $\cos\theta_1 = \sqrt{1 - \left(\dfrac{n_0}{n_1}\right)^2 \sin^2\theta_0}$.

5   The present invention also concerns the characteristics which will appear in the following description and which should be considered individually or according to all their technically possible combinations:

- the values of the refraction index n_1 and of the thickness e_1 of the
10   layer are within 2 % such that :

$$\sigma = 0$$

- the substrate and the layer are dielectric or little absorbent, the module of the imaginary portion of their complex index being smaller than
15   0.01, the general conditions being reduced to the conditions :

$$n_1 e_1 \cos\theta_1 = \frac{\lambda}{4} + k\frac{\lambda}{2}$$

and

$$n_1^2 = \frac{n_2^2 + \sqrt{n_2^2 \cos^2\theta_0 (n_2^2 - n_0^2 \sin^2\theta_0)}}{n_2^2 + n_0^2 \cos^2\theta_0}$$

20   with k integer and with an uncertainty of 2 % on the values of n_1 and e_1 ;

- θ_0 is smaller than 5°, the general conditions being reduced to

$$\frac{2}{n_1^2} = \frac{1}{n_0^2} + \frac{1}{n_2^2}$$

25   and

$$n_1 e_1 \cos\theta_1 = \frac{\lambda}{4} + k\frac{\lambda}{2}$$

with k integer and with an uncertainty of 2 % on the values of n_1 and e_1 ;

- the support is intended for use with annular incident illumination with an angle of incidence θ_0 which is unique within $\pm 2.5°$;

- the support is intended for use in incident and convergent axial illumination with an average angle of incidence θ_0 associated with its total angular opening $\Delta\theta_0$ by the relation:

$$\cos\theta_0 = \cos^2\left(\frac{\Delta\theta_0}{2}\right)$$

- the illumination is monochromatic or quasi-monochromatic at the wavelength λ;

- the illumination has a continuous wide spectrum or polychromatic with maximum span $\pm 0.3 \lambda$ around its average wavelength λ;

- the support being intended for use in the air as a surrounding medium, with $\theta_0 = 30°$ and $\lambda = 589.3$ nm, the substrate is made of cadmium with $n_2 = 1.13-5.01j$, the layer having an index $n_1 = 1.42$ and $e_1 = 1084$ Angströms;

- the substrate and the layer have the specificities of the following table wherein n_1 and e_1 are the index and the thickness of the layer, n_2 the complex refraction index of the substrate, in the air as a surrounding medium, $\theta_0 = 5°$ and $\lambda = 540$ nm

Substrate	n_2	n_1	e_1(Å)
Gold	$0.40 - 2.6j$	1.70	694
Silver	$0.13 - 3.44j$	1.59	795
Aluminium	$0.92 - 0.95j$	2.01	346
Nickel	$1.76 - 3.2j$	1.51	847

- θ_0 is an average angle of incidence equal to 20° and the substrate and the layer have the specificities of the following table wherein n_1 and e_1 are the index and the thickness of the layer, n_2 the complex refraction index of the substrate, in the air as a surrounding medium (3) and $\lambda = 540$ nm

Substrate	n_2	n_1	e_1(Å)
Gold	$0.40 - 2.6j$	1.64	739
Silver	$0.13 - 3.44j$	1.55	838
Aluminium	$0.92 - 0.95j$	1.89	399
Nickel	$1.76 - 3.2j$	1.48	890

- θ_0 is equal to 5° and the substrate and the layer have the specificities of the following table wherein n_1 and e_1 are the index and the thickness of the layer within 2 %, n_2 the complex refraction index of the substrate, n_0 the index of the surrounding medium, λ= 589.3 nm when the layer is made of cadmium and λ= 540 nm in the other cases

Substrate	n_2	n_0	n_1	e_1
Gold	$0.40 - 2.6j$	1.33	2.42	490
Gold	$0.40 - 2.6j$	1.5	1.79	755
Silver	$0.13 - 3.44j$	1.33	2.28	512
Silver	$0.13 - 3.44j$	1.5	2.7	412
Aluminium	$0.92 - 0.95j$	1	1.89	399
Nickel	$1.76 - 3.2j$	1.33	2.11	572
Nickel	$1.76 - 3.2j$	1.5	2.45	473
Cadmium	1.13-5.01j	1	1.49	970
Cadmium	1.13-5.01j	1.33	2.05	684
Cadmium	1.13-5.01j	1.5	2.36	582
Tin	1.48-5.25j	1	1.48	899
Tin	1.48-5.25j	1.33	2.02	640
Tin	1.48-5.25j	1.5	2.33	548
Copper	1.04-2.59j	1	1.62	746
Copper	1.04-2.59j	1.33	2.23	423
Copper	1.04-2.59j	1.5	2.83	351
Iron (evaporated)	1.51-1.63j	1	1.54	737
	1.51-1.63j	1.33	2.23	423
	1.51-1.63j	1.5	2.72	305

- the parameters defined by the claims 10 to 14 are kept with the exception of the wavelength λ and of the thickness e_1 of the layer 2 which are modified proportionally, $\frac{e_1}{\lambda}$ not being modified.

5   The invention also concerns an accessory intended for observing a preferably liquid sample formed of a Petri dish and of a support intended for receiving said sample, the support being the bottom of this dish.

The invention also concerns the devices having the following
10   characteristics:
- a device for observing a sample including an optical microscope, a support intended for receiving said sample and two crossed polarisers;
- a device for observing a sample including an optical microscope, an accessory intended for receiving said sample and two crossed polarisers;
15   - a device for observing a sample including an optical microscope, a support intended for receiving said sample, a polariser and a quarter-wave plate;
- a device for observing a sample including an optical microscope, an accessory intended for receiving said sample, a polariser and a quarter-wave
20   plate;
- a device for observing a sample whereof the optical microscope is fitted with a differential interferential contrast device.

In different possible embodiments, the invention will be described more
25   in detail with reference to the appended drawings wherein:
- Figure 1 is a schematic representation of the oblique transmission and reflection of a planar wave at the planar interface between two semi-infinite media 0 and 1 ;
- Figure 2 is a schematic representation of a support, according to the
30   invention.

To describe completely the invention, the support that it concerns is presented, thereafter, in its context while using simultaneously the definitions

of the elements involved and the associated devices with respect to which they should be situated.

I. Conventions:

5   The sign conventions adopted are those represented to Figure 1 and described also in the book of Azzam and Bashara entitled "ellipsometry and polarised light", North-Holland, p 271.

This figure represents the oblique transmission and reflection of a planar wave at the planar interface between two semi-infinite media 0 and 1. θ_0 and θ_1 are

10   respectively the angles of incidence and of refraction. p and s are respectively the polarisations parallel and perpendicular to the plane of incidence. The vector generated $\hat{p} \times \hat{s}$ of the unit vectors along these axes is parallel to the direction of propagation and oriented in the direction of the propagation.

15 ### II. Definitions:

- By "support" is meant the association of a solid substrate 1 of complex index of refraction n_2 and of a coating including at least one layer 2 of thickness e_1 and of complex refraction index n_1. This support is placed in a surrounding medium 3 of index n_0.

20 - By "sample" is meant the association of the object-carrying support and of the object it carries, said support being the object of the invention.

- By "observation" is meant a direct ocular observation through a instrument or the acquisition of an image or of a signal by detection means including a recording device such as an analog or digital camera, a CCD array or a

25   measuring device such as a detector (photovoltaic cell, photomultiplier) or a matrix of detectors (array of photodiodes, CCD, ...) placed in a plane where the image of the sample is formed.

- A perfectly reflecting support is defined by the relations

$$|r_p| = |r_s| = 1 \qquad\qquad (E1)$$

30   on its Fresnel coefficients r_p and r_s.

- "The standardised intensity" of the image of a support or of a sample obtained by means of an imaging device operating in reflection and including possibly one or several polarising elements and/or modifying the polarisation of the light and collected by means of detection or of

observation is defined by the ratio $R = \dfrac{I}{I_0}$ where $I = I(x, y)$ designates the intensity collected at one point *(x, y)* of the image and where $I_0 = I_0(x, y)$ designates the intensity collected at one point *(x, y)* of the image by the same means of detection or of observation adjusted similarly failing any
5   polariser and while using a perfectly reflecting support.

- An "antiglare" (AR) support is defined as having a minimal coefficient of reflection on the set of its adjustable parameters. It relates to the conditions of observation or of measurement. An ideal antiglare support is such that its coefficient of reflection is nil.

10   • A "contrast amplifying" support is defined so that the object that it carries is observed with a contrast made maximal or so that a physical quantity of the object is measured with a sensitivity made maximal on the set of its adjustable parameters. It is also relative to the conditions of observation or of measurement. It is moreover relative to the nature of the object

15   observed. When the nature of the object is not specified, the object will be defined by default as a thin film of thickness 1 *Angström* (Å) and of index n identical to that of the surface of the support, i.e. of the material forming the layer 2 in contact with the object.

20   **III. Illumination conditions:**

"The convergent axial illumination" is convergent and with radial symmetry around the normal to the surface, defined as "the axis" below in the description, with an opening angle θ_{max}. The angles of incidence partaking of the illumination cone are therefore all the angles ranging between 0 and θ_{max}.

25   « The annular illumination » also possesses a radial symmetry around the normal to the surface, i.e. the axis, but it is defined by a single angle of incidence θ better than within 5 degrees. The "anisotropic" illumination is defined in that a radial symmetry of the azimuths φ is broken, but the symmetry with respect to the axis kept, and finally "the oblique illumination" is

30   defined by a single angle of incidence θ and a single azimuth φ.

Except when specified, the illumination will be considered as annular in the remainder of the description. However, the supports described for use under annular illumination with an angle of incidence θ_0 are also intended for use with convergent axial illumination with an average angle of incidence θ_0 .

When the illumination is convergent axial, by "average angle" θ_0 is meant the angle defined by the circle separating the illuminating cone in two solid angles of the same value. The support is therefore intended for use under convergent illumination with angular opening $\Delta\theta_0$ and centred on θ_0,

5 where θ_0 is the average angle of incidence ranging 0 and $\Delta\theta_0$ defined by the relation $\cos\theta_0 = \cos^2\left(\dfrac{\Delta\theta_0}{2}\right)$ (Figure 2).

By default, the collection of the light by the means of detection or of observation is supposed with radial symmetry around the normal to the surface, with the same unique angle of collection or the same opening angle

10 as the illumination, in the remainder of the description.

The illumination is spatially incoherent, which means that a light beam partaking of the illumination may only interfere with itself. Under these conditions, the contributions to the formation of the image must be added in amplitude along a beam and in intensity on the set of beams, i.e. on the set of

15 angles of incidence θ and of the azimuths φ contributing to the illumination. The Fresnel coefficients r_p and r_s are complex functions of θ. In the case of anisotropic supports, these are moreover functions of φ. But the support is supposedly isotropic in the remainder of the description excepted when specified.

20 Similarly, the illumination is supposedly monochromatic or quasi-monochromatic, the light beam being centred on a wavelength λ. However, the supports described for use under quasi-monochromatic illumination at the wavelength λ are also intended for use with a white light or polychromatic illumination, the spectrum of the illumination being then centred on λ.

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IV. Conditions of polarisation

IV.1) Non-polarised light:
The standardised intensity R_{NP} is written:

30
$$R_{NP} = \frac{1}{2}\left(|r_p|^2 + |r_s|^2\right) \qquad (E2)$$

It is equal to 1 for a perfectly reflecting support.
The reflection coefficient of the support is defined by this standardised intensity in non-polarised light. It depends on the illumination conditions.

IV.2) Polarised light:

The sample is placed between a first and a second polarisers. In a preferred embodiment and for the remainder of the description, said polarisers will be considered as linear. When said sample is illuminated by a light beam, the beam traverses on its optical path the first polariser which defines its polarisation, then after interaction with the sample, the light beam traverses the second polariser. The first polariser is called "illumination polariser". The second polariser is the analysis polariser or "analyser". The first and the second polarisers form together an angle ϕ modulo π.

The standardised intensity $R(\phi)$ is written:

$$4\,R(\phi) = \cos^2\phi\,(|r_p|^2 + |r_s|^2) - \frac{\cos 2\phi}{4}\,|r_p + r_s|^2 \quad (E3)$$

or, similarly:

$$4\,R(\phi) = \frac{1}{2}(|r_p|^2 + |r_s|^2) + \frac{\cos 2\phi}{4}\,|r_p - r_s|^2 = R_{NP} + \frac{\cos 2\phi}{4}\,|r_p - r_s|^2 \quad (E4)$$

In the particular case where le first and second polarisers are parallel, it reads:

$$4\,R(0) = (|r_p|^2 + |r_s|^2) - \frac{1}{4}\,|r_p + r_s|^2 = 2R_{NP} - \frac{1}{4}\,|r_p + r_s|^2 \quad (E5)$$

or, similarly:

$$4\,R(0) = \frac{1}{2}(|r_p|^2 + |r_s|^2) + \frac{1}{4}\,|r_p - r_s|^2 = R_{NP} + \frac{1}{4}\,|r_p - r_s|^2 \quad (E6)$$

In the particular case where the first and second polarisers are perpendicular, i.e. crossed, it reads:

$$4\,R(\tfrac{\pi}{2}) = \frac{1}{4}\,|r_p + r_s|^2 \quad (E7)$$

or, similarly:

$$4\,R(\tfrac{\pi}{2}) = \frac{1}{2}(|r_p|^2 + |r_s|^2) - \frac{1}{4}\,|r_p - r_s|^2 = R_{NP} - \frac{1}{4}\,|r_p - r_s|^2 \quad (E8)$$

In the particular case where both polarisers form together an angle $\frac{\pi}{4}$, it reads:

$$4\,R(\tfrac{\pi}{4}) = \frac{1}{2}(|r_p|^2 + |r_s|^2) = R_{NP} \quad (E9)$$

Thus, the standardised intensity may be modulated around its average value $R_{NP}/4$ with an amplitude $\frac{1}{16}\left|r_p - r_s\right|^2$ by relative rotation of the analyser and of the illumination polariser.

5   If one considers $R^+ \equiv \frac{1}{4}\left|r_p + r_s\right|^2$ and $R^- \equiv \frac{1}{4}\left|r_p - r_s\right|^2$.

Then:

$$4R(\phi) = 2R_{NP}\cos2\phi - R^+\cos2\phi = R_{NP} + R^-\cos2\phi = R^+ + 2R^-\cos2\phi \quad \text{(E3-E4 bis)}$$

$$4R(0) = 2R_{NP} - R^+ = R_{NP} + R^- \quad \text{(E5-E6 bis)}$$

10

$$4\,R(\tfrac{\pi}{2}) = R^+ = R_{NP} - R^- \quad \text{(E7-E8 bis)}$$

The support of the invention is intended for use between a first and a second crossed polarisers. The light beam incident on the support is therefore
15   polarised.

V. Antiglare supports and coatings

V.1) Non-polarised light.
20   It is known that in non-polarised light, an antiglare support (AR) placed in a surrounding medium of index n_0 exhibits a coefficient R_{NP} which is minimum. The ideal antiglare support, which verifies $R_{NP} = 0$ may only be obtained for a single angle of incidence $\theta_0 = 0$ and may only be realised on a substrate 1 of index n_2 different of n_0 where n_2 is the complex refraction index
25   of the substrate 1, only by applying a single layer 2 of optical thickness $\frac{\lambda}{4}$ and of optical index n_1 so that:

$$n_1 = \sqrt{n_0 n_2} \quad \text{(E10)}$$

n_0 being the optical index of the incident medium.
30   The optical thickness $\frac{\lambda}{4}$ means that the physical thickness e_1 of the layer 2 is linked with the wavelength λ of the light beam enabling the illumination and

with the direction θ_1 of said beam with respect to the normal to the surface after refraction in the layer 2, or refracted angle of incidence, by the relation:

$$n_1 e_1 \cos\theta_1 = \frac{\lambda}{4} + k\frac{\lambda}{2} \qquad \text{(E11)}$$

5   with k integer and $\cos\theta_1 = \sqrt{1 - \left(\frac{n_0}{n_1}\right)^2 \sin^2\theta_0}$.

In the case considered, this relation is therefore reduced to:

$$n_1 e_1 = \frac{\lambda}{4} + k\frac{\lambda}{2} \qquad \text{(E11 bis)}$$

Let us remind that the refracted angle θ_1 is linked with the incident angle θ_0 on
10   the support by the Snell relation: $n_1 \sin\theta_1 = n_0 \sin\theta_0$ where n_0 is the optical index of the incident medium. Similarly, we have $n_2 \sin\theta_2 = n_0 \sin\theta_0$. Let us remind that $\sin^2\theta_i + \cos^2\theta_i = 1$ in all the media.

In practice, it is often preferable to use supports AR which are ideal but
15   little sensitive to the wavelength λ or to the angle of incidence θ_0 of the incident light beam. These supports are obtained by multilayer stacks called anti-reflection treatments. In an embodiment, these treatments are used for eliminating the reflections on spectacle glasses or for eliminating the spurious light generated by the reflections on the dioptres of the optical systems.
20   There exist however other cases where the supports AR, more selective and more efficient, are sought after, for example, for preparing transmission narrow pass-band interferential filters.

V.2) Polarised light supports and coatings "AR-Pol"
25   The invention concerns supports similar to the antiglare supports, but intended for use in polarised light. They differ from antiglare supports by their composition and by their optical properties. We shall designate it as "AR-Pol".

These new supports form a set which even wider and more diversified than the conventional antiglare supports, and may intervene as accessories or
30   as components in numerous methods or devices. This set is the first object of the present invention. It is subdivided in families corresponding to groups by nature or by function whereof we shall give here a few examples. Each family

is designated as "AR-( )-Pol-( )-( )... ", where the successive brackets represent the additional precisions enabling to define it.

From equations E2 to E9 can be derived the following order relations:

5

$$R(\tfrac{\pi}{2}) \le R(\phi) \qquad (I1)$$

For any ϕ, and:

$$4\,R(\tfrac{\pi}{2}) \le 4\,R(\tfrac{\pi}{4}) = R_{NP} \le 4R(0) \qquad (I2)$$

10   As shown by the inequalities I1 and I2, the standardised intensity of the image of a support or of an isotropic sample is always smaller between two crossed polarisers than for any other relative orientation of the polarisers and in the absence of polariser.

15   **V.3) Polarising microscope:**

As it is known, the extinction coefficient C_e of a polarising microscope, is one of its important technical characteristics. It must also be as small as possible. In the case of a reflection-operating microscope, it is defined as the ratio of the intensities reflected by a perfectly reflecting support placed on the
20   one hand between a first and a second crossed polarisers and on the other hand, between a first and a second parallel polarisers, i.e.:

$$Ce = \frac{I_0\left(\pi/2\right)}{I_0(0)} \qquad (E14)$$

25   The examination of the equations E5 and E7 shows that the definition of the perfectly reflecting support given by the equations E1 is not sufficient since the intensity collected $I(\phi)$ depends on the relative phases of both Fresnel coefficients r_p and r_s through their sum. As the purpose of the extinction coefficient is to characterise the microscope properly speaking, it is
30   necessary to introduce more precision in the definition of the perfectly reflecting support while considering either the virtual condition $r_p = r_s = 1$, or the condition $r_p = -r_s = 1$. The first condition not being physically realistic, it is therefore eliminated to keep only the second. Moreover, one always has $r_p = -$

r_s for an angle of incidence of the light beam on the support so that $\theta_0 = 0$, which enables to associate the coefficient $Ce = 0$ to the ideal microscope.

We define the *Useful extinction coefficient* of the set formed by the microscope and the sample as the ratio:

5

$$C_u = \frac{I\left(\pi/2\right)}{I(0)} \qquad (E15)$$

the sample being at least formed of a support.

For a reflection-operating polarising microscope, the value of C_u is
10   derived directly from the equations E6 and E8:

$$C_u = \frac{R_{NP} - \frac{1}{4}\left|r_p - r_s\right|^2}{R_{NP} + \frac{1}{4}\left|r_p - r_s\right|^2} = \frac{R_{NP} - R^-}{R_{NP} + R^+} \qquad (E16)$$

Which also reads:

15

$$C_u = \frac{\frac{1}{4}\left|r_p + r_s\right|^2}{2R_{NP} - \frac{1}{4}\left|r_p + r_s\right|^2} = \frac{R^+}{R_{NP} - R^+} \qquad (E16 \text{ bis}),$$

since:

20    $$R_{NP} = \frac{1}{4}\left|r_p + r_s\right|^2 + \frac{1}{4}\left|r_p - r_s\right|^2 = R^+ + R^- \quad (E16 \text{ ter})$$

Maximisation of the extinction coefficient:

The ideal antiglare support corresponds to $R_{NP} = 0$. According to the equation E2, this implies simultaneously $r_p = 0$ and $r_s = 0$, and therefore $R(\phi) =$
25 · 0 regardless of the value of ϕ. The useful extinction coefficient is not defined any longer. One may then, by extension, define it as:

$$C_u = \lim_{R_{NP} \to 0} Cu \qquad (E17),$$

which enables to include the limit case of the ideal antiglare support in the following discussion.

According to the equation E16 bis, the useful extinction coefficient C_u is an increasing function of the ratio $\dfrac{|r_p + r_s|^2}{R_{NP}}$. To improve the extinction

5   coefficient, and therefore to reduce it, it is this ratio which should be minimised.

VI. Antiglare supports between crossed polarisers AR-X-Pol

We shall define the family of the supports AR-X-Pol as the family of the

10   supports for which the ratio $\dfrac{|r_p + r_s|^2}{R_{NP}}$ is made minimal, which represents a rule

for designing said support. Similarly, we shall give the following functional definition: it is the family of the supports which optimise the useful extinction coefficient of the set [microscope + support] for an ideal polarising microscope.

15   The ideal supports AR-X-Pol are obtained when this minimum value is nil. They are therefore given by the condition $\sigma = 0$.

We shall define the coating AR-X-Pol as the layer 2 which enables to transformer a given substrate 1 in a support AR-X-Pol.

20   VI.1) Supports AR-X-Pol-SD-1D

These supports correspond to the case where the materials used for the substrate 1 and the layer 2 are non absorbent, which means that the modulus of the imaginary portion of their index is smaller than 0.01.

The expression of r_p and r_s for a solid of optical index n_2 covered with a

25   single layer 2 of index n_1 and of thickness e_1 in a surrounding medium of index n_0 is conventionally given by:

$$r_m = \frac{r_{01(m)} + r_{12(m)}e^{(-2j\beta_1)}}{1 + r_{01(m)}r_{12(m)}e^{(-2j\beta_1)}} \qquad (E17)$$

[Azzam and Bashara, "ellipsometry and polarised light", North-Holland, 1987], with: either $m = s$, or $m = p$, according to the polarisation considered and with:

30

$$\beta_1 = \frac{2\pi n_1 e_1 \cos\theta_1}{\lambda} \qquad (E18)$$

. where $\cos\theta_1 = \sqrt{1 - \left(\frac{n_0}{n_1}\right)^2 \sin^2\theta_0}$.

This equation enables to read:

5

$$\sigma = r_p + r_s = \frac{\sigma_{01} + \sigma_{12}(1+\pi_{01})e^{(-2j\beta_1)} + \sigma_{01}\pi_{12}e^{(-4j\beta_1)}}{(1 + r_{01(p)} + r_{12(p)}e^{(-2j\beta_1)})(1 + r_{01(s)}r_{12(s)}e^{(-2j\beta_1)})} \quad \text{(E19)}$$

where σ_{ij} and π_{ij} represent respectively the sum and the product of the Fresnel coefficients:

$$r_{ij(p)} = \frac{n_j\cos\theta_i - n_i\cos\theta_j}{n_j\cos\theta_i + n_i\cos\theta_j} \quad \text{(E20)}$$

10   and

$$r_{ij(s)} = \frac{n_i\cos\theta_i - n_j\cos\theta_j}{n_i\cos\theta_i + n_j\cos\theta_j} \quad \text{(E21)}$$

VI.1-1) Ideal supports AR- X-Pol- SD-1D

These supports are obtained when $\sigma = 0$ (E19 bis)

15   Contrary to the condition [$R_{NP} = 0$] which defines the ideal conventional antiglare supports, the condition $\sigma = 0$ is always verified in normal incidence. The cases wherein this condition is also verified for a non-zero angle of incidence will be examined below.

The materials considered being dielectric or quasi-dielectric, $r_{ij(p)}$ and 20 $r_{ij(s)}$ are real, and therefore σ_{ij} and π_{ij} are real. The situation sought after corresponds therefore to the case where $e^{-2j\beta_1}$ is real, which implies, either:

$$2\beta_1 = (2k + 1)\pi \quad \text{(E22)}$$

or:

25    $$2\beta_1 = 2k\pi \quad \text{(E23)}$$

with k integer, i.e. either:

$$n_1e_1\cos\theta_1 = \frac{\lambda}{4} + k\frac{\lambda}{2} \quad \text{(E24)}$$

30

or:

$$n_1 e_1 \cos\theta_1 = k\frac{\lambda}{2} \quad \text{(E25)}$$

5   The condition E24 is identical to the condition E11 which is one of the relations defining the conventional antiglare supports. One has therefore $e^{-2j\beta_1} = \pm 1$. Looking for solutions of the equation (E19 bis) is reduced to solving the equations:

10                          $$\sigma_{01}(1 + \pi_{12}) - \sigma_{12}(1 + \pi_{01}) = 0 \quad \text{(E26)}$$
and:

$$\sigma_{01}(1 + \pi_{12}) + \sigma_{12}(1 + \pi_{01}) = 0 \quad \text{(E27)}$$

It should be noted $c_k^2 = \cos^2\theta_k$ and $s_k^2 = \sin^2\theta_k$ for all the media.

15

Let us remind the Snell relation: $n_k\sin\theta_k = n_0\sin\theta_0$
It is easy to establish:

$$\sigma_{ij} = \frac{2n_i n_j (c_i^2 - c_j^2)}{n_i n_j (c_i^2 + c_j^2) + c_i c_j (n_i^2 + n_j^2)} \quad \text{(E28)}$$

20

and:

$$1 + \pi_{ij} = \frac{2n_i n_j (c_i^2 + c_j^2)}{n_i n_j (c_i^2 + c_j^2) + c_i c_j (n_i^2 + n_j^2)} \quad \text{(E29)}$$

hence:

$$\frac{\sigma_{ij}}{1 + \pi_{ij}} = \frac{(c_i^2 - c_j^2)}{(c_i^2 + c_j^2)} \quad \text{(E30)}$$

25   The equation E26 is thus reduced to:

$$c_0 c_2 = c_1^2 \quad \text{(E31)}$$

which reads then:

30        $$2n_0^2 n_2^2 - n_1^2 (n_0^2 + n_2^2) = (n_0^2 n_2^2 - n_1^4)\sin^2\theta_1 \quad \text{(E32)}$$

This equation may be solved without any difficulty with respect to each of its parameters.

It is never satisfied with a conventional antiglare support.

In the case where the angle of incidence is very small, this condition is reduced to:

$$\frac{2}{n_1^2} = \frac{1}{n_0^2} + \frac{1}{n_2^2} \qquad \text{(E33)}$$

The couple of equations E25 and E27 has no solution for $n_0 \neq n_2$.

Both formulas E24 and E32 (or similarly E35 bis) define the family of ideal supports AR-X-Pol-SD-1D for a random angle of incidence defined indifferently by its value θ_0 in the surrounding medium (3) or by its refracted value θ_1 in the layer (2) and also for a convergent axial illumination defined by an average refracted angle $<\theta_1>$.

Both formulas E24 and E33 define the sub-family of ideal supports AR-X-Pol-SD-1D for an optical instrument with an annular illumination defined by a refracted angle of incidence θ_1 or with a convergent axial illumination with small opening and defined by an average refracted angle $<\theta_1>$.

Properties of the equations E32 and E33 and consequences:

The 4 variables n_0, n_2, n_1 and $\sin\theta_1 = \frac{n_0}{n_1} \sin\theta_0$ are not independent. It

suffices to consider the reduced variables $x = \left(\frac{n_1}{n_0}\right)^2$ and $y = \left(\frac{n_2}{n_0}\right)^2$ to

describe all the situations.

The equation E32 then becomes:

$$x^2 (y + c_0^2) - 2xy + ys_0^2 = 0 \qquad \text{(E34)}$$

whereof the solutions are given by
i) the solutions in x:

$$x = \frac{y + c_0\sqrt{y(y - s_0^2)}}{y + c_0^2} \qquad \text{(E35)}$$

which can also read:

$$n_1^2 = \frac{n_2^2 + \sqrt{n_2^2 \cos^2 \theta_0 (n_2^2 - n_0^2 \sin^2 \theta_0)}}{n_2^2 + n_0^2 \cos^2 \theta_0} \qquad \text{(E35bis)}$$

ii) the solutions in y:

$$y = \frac{c_0^2 x^2}{c_0^2 - (x-1)^2} \qquad \text{(E36)}$$

iii) the solutions in θ_0:

$$c_0^2 = \frac{y(x-1)^2}{y - x^2} = \frac{n_2^2 (n_1^2 - n_0^2)^2}{n_0^2 (n_0^2 n_2^2 - n_1^4)} \qquad \text{(E37)}$$

iv) the variation of x with the angle of incidence θ_0:

$$\frac{dx}{d\theta_0} = -\sin 2\theta_0 \frac{2y(x-1)(y-x)}{(y-x^2)^2} = -\sin 2\theta_0 \frac{2n_0^2 n_2^2 (n_1^2 - n_0^2)(n_2^2 - n_0^2)}{(n_0^2 n_2^2 - n_1^4)^2} \qquad \text{(E38)}$$

which shows that the optimal index n_1 of the layer 2 decreases when the angle of incidence increases.

For small incidences, the evolution of n_1 with θ_0 is described by the approximation:

$$x \approx \frac{2y}{(1+y)}\left(1 - \frac{(y-1)^2}{4y(y+1)}\theta_0^2\right) = \frac{2n_2^2}{(n_0^2 + n_2^2)} \qquad \text{(E39)}$$

i.e.:

$$n_1^2 \approx \frac{2n_0^2 n_2^2}{(n_0^2 + n_2^2)} - n_0^2 \left(\frac{n_2^2 - n_0^2}{n_2^2 + n_0^2}\right)^2 \frac{\theta_0^2}{2} \qquad \text{(E40)}$$

which shows again that the optimal index decreases when the angle of incidence increases.

This dependence is the smaller as y is close to 1.

Still, for a perfect microscope ($C_e = 0$), the useful extinction coefficient is always nil in normal incidence ($C_u = 0$).

An optimised support AR-X-POL for a non-zero angle, for example of the order of 20 degrees, is optimal for working with a corresponding annular illumination. But moreover, since it enables to obtain $C_u = 0$ simultaneously for a zero incidence and for a non-zero incidence, it enables to keep an excellent useful extinction coefficient on the set of the opening cone of a convergent axial illumination, from 0 to 30 degrees for instance. This constitutes a

considerable advantage of the coatings AR-X-POL on the conventional antiglare coatings for all the applications of the antiglare supports compatible with the use of a polarised light.

This flexibility on the angle of incidence translates by an analogue flexibility on the wavelength of the small incidence illumination. Indeed, by logarithmic differentiation of the equation E24 for instance, we obtain:

$$-\tan\left(\theta_1\right)\Delta\theta_1 = \frac{\Delta\lambda}{\lambda}$$

which shows that a variation in wavelength as high as 30% with an annular illumination is equivalent to an opening of 30 degrees on a convergent illumination with a fixed wavelength. This variation in wavelength covers the whole visible spectrum around $\lambda = 0.55\mu m$.

If, on the contrary, it is desirable to exploit the colouring effects for detection purposes, the use of an annular illumination with high angle of incidence enables to bring high sensitivity to the wavelength.

The supports and coatings AR-X-Pol are therefore particularly advantageous for uses in white light.

The index n_1 of the layer 2 is intermediate between the index n_2 of the substrate 1 whereon it is formed and the index n_0 of the surrounding medium. The equations E32 and E33 are symmetrical in n_0 and n_2.

This shows that said layer 2 possesses the same properties as the light reflecting from the small index medium to the high index medium or from the high index medium to the mall index medium.

Thus, one may extinguish the reflection of a polarised light when the substrate 1 is the end of an optical fibre, or a blade or the bottom of a Petri dish when observed from beneath on a reversed microscope.

VI.1-2) Approximate supports AR-Pol-X-SD-1D

When the three indices (n_0, n_1 and n_2) are imposed or constrained, the minimum of $|\sigma|$ with respect to e_1 is not nil if the condition E32 is not verified, but this minimum exists nevertheless and corresponds to the best possible extinction taking the constraints into account.

The best possible extinction between crossed polarisers is thus obtained by seeking the minimum of $|\sigma|$ (or similarly that of $|\sigma|^2$) with respect to e_1. With θ_1 and n_1 fixed, this amounts to looking for it with respect to β_1.

Let us assume $z = e^{-2j\beta_1} = \cos2\beta_1 - j\sin2\beta_1$. The expression of σ given by the equation E19 is the ratio of 2 polynomials of degree 2 in z. Since $|z| = 1$, the square of the modulus of each of these polynomials only contains constant terms, $\cos2\beta_1$ terms and $\cos4\beta_1$ terms. The derivate of their ratio is therefore proportional to $\sin2\beta_1$. The solutions of $\sin2\beta_1 = 0$ are therefore solutions of the equation $\dfrac{d|\sigma|^2}{d\beta_1} = 0$. One finds therefore the solutions given by the equations E24 and E25. Thus, even when the conditions E32 or E33 are not satisfied, one may optimise the thickness of the layer 2 to optimise the useful extinction coefficient under a polarising microscope, and this optimum value corresponds either to a layer 2 " at $\dfrac{\lambda}{4}$ ", or to a layer 2 " at $\dfrac{\lambda}{2}$ ".

VI.2) Supports AR-X-Pol-1

We shall describe here the rules for building supports **AR-X-Pol** formed of a solid substrate 1 covered with a single layer 2 in the general case where the support, the layer 2, and the incident medium 3 have any complex optical indices complexes (possibly absorbent media).

VI.2-1) Ideal supports AR-X-Pol-1

Among these, the ideal supports for the angle θ_0 are obtained when $\sigma = 0$, where the quantity σ is given by the equation E19, where β_1 is linked with the angle θ_1 by the relation E18, θ_2, θ_1 and θ_0 being always linked with the Snell relation extended to the complex functions.

The equation:

$$\sigma_{01} + \sigma_{12}(1 + \pi_{01})z + \sigma_{01}\pi_{12}z^2 = 0 \qquad \text{(E19 ter)}$$

always has 2 solutions z_1 and z_2 arranged according to their modulus, $|z_1| < |z_2|$, which are expressed in relation to the coefficients σ_{01}, σ_{12}, π_{01} and π_{12} themselves in relation to the three indices n_0, n_1, n_2 and to the angle θ_0.

The medium 1 not being amplifying, the solution z_1 is the sole acceptable. Its expression translates by a relation which defines digitally the family of the ideal supports AR-X-POL-1. This family is delineated by the condition $|z_1| \leq 1$.

5   **VI.2-2) Approximate supports AR-X-Pol-1.**

When the three indices are imposed or constrained and one of them at least is complex, the minimum of $|\sigma|$ with respect to e_1 is not zero any longer if the condition E42 is not verified by the solution of smaller modulus of the equation E19ter, but it exists nevertheless since $|\sigma|$ is a quasi-periodic

10   function of e_1. By "quasi-periodic" is meant here that n_1 being complex, the modulus of $e^{-4j\beta 1}$ is dampened when e_1 increases.

The best possible extinction coefficient taking the constraints into account is obtained by looking for the minimum of $|\sigma|$ (or similarly that of $|\sigma|^2$) with respect to e_1. At θ_1 and n_1 fixed, this amounts to looking for it with respect

15   to the complex quantity β_1, which may be done numerically.

The thickness e_1 is then given by the equation:

$$e_1 = \frac{\lambda \beta_1}{2\pi |n_1 \cos \theta_1|} \qquad \text{(E43)}$$

which is a generalisation of the equation E18.

20

VI.2- 3) Ideal dielectric coating on absorbent support.

A useful particular case is that where only the support is absorbent, the indices of the other media remaining real. Then β_1 is real, and $|\sigma|$ is a periodic function of β_1.

25   But as σ_{12} and π_{12} are complex, $e^{-2j\beta 1}$ is not real. The optimal thickness which negates $|\sigma|$ is therefore given by:

$$n_1 e_1 \cos \theta_1 = e_c + k\frac{\lambda}{2} \qquad \text{(E44)}$$

where the smallest of the solutions e_c is not equal to $\frac{\lambda}{4}$ nor to $\frac{\lambda}{2}$ any longer.

This equation generalises the equations E24 and E25.

30   Thus, in the case of a substrate 1 absorb, the coating AR-X-POL differs from a conventional antiglare coating, not only by its index, but also by its thickness.

VII. Ideal contrast amplifying supports

For visualising the edge of a study object having the form of a thin film placed on the surface of the support, it is advisable to exploit the difference between the intensities collected on the one hand while observing the film and on the other hand while observing the bare surface of the substrate 1 which are noted I_F and I_S (or similarly $I(F)$ and $I(S)$). These intensities are proportional to the corresponding standardised intensities.

The contrast of the edge of the film is given by the following relation:

$$C_f = \frac{I_F - I_S}{I_F - I_S} \qquad (E50)$$

I_F and I_S being positive, C_f is a strictly increasing function of the ratio $\frac{I_F}{I_S}$.

To visualise the film correctly, $|C_f|$ should be maximised and therefore the ratio $\frac{I_F}{I_S}$ should be made maximal ($I_S \rightarrow 0$, to tend towards a contrast of 1) or minimal ($I_F \rightarrow 0$, to tend towards a contrast of -1). Therefore, either the surface or the film should be extinguished.

A sensitive visualisation process lies on the one hand on a good extinction, and on the other hand on a critical extinction, i.e. very sensitive to the thickness of the last layer 2 of the stack. The antiglare supports AR-X-Pol exhibit these qualities and are therefore also contrast amplifying supports.

The performances of a visualisation process may be quantified by the contrast obtained when the film observed becomes extremely thin. In this case, I_F and I_S become neighbours and $dI = I_F - I_S$ is close to a differential element.

For a film of very small thickness Δe placed on the support, one may write in the first order in Δe:

$$\frac{I_F}{I_S} = 1 + \frac{1}{I_s}\frac{dI}{de}\,\Delta e \approx 1 + \Delta e\,\frac{d}{de}\ln I \qquad (E51)$$

where it has been assumed that optical index of the film is identical to that of the upper layer 2, i.e. of the last layer 2 of the stack, and where $\frac{dI}{de}$ is the

derivate of the intensity reflected by the bare substrate 1 with respect to the thickness e of this layer 2.

In the case where the substrate 1 is composed of a solid support covered with a single dielectric layer 2, e is therefore the thickness of the single layer 2. The film appears therefore as a simple fluctuation of thickness of the upper layer 2.

An optimal contrast is therefore obtained for both situations:

i) $\dfrac{d}{de} \ln I \rightarrow +\infty$ (extinction of the support, $C_f = +1$)

ii) $\dfrac{d}{de} \ln I = -1$ (extinction of the film, $C_f = -1$)

The optimal contrast may only be reached with a total extinction.

The sensitivity of the visualisation is, in *Angströms*$^{-1}$:

$$\frac{C_f}{\Delta e} = \frac{1}{2} \frac{d \ln I}{de} \qquad \text{(E52)}$$

It has a meaning only when the contrast is small (when $\dfrac{d \ln I}{de} \Delta e$ is very small compared to 1) and enables to compare detection thresholds.

VII.1) In non-polarised light

In the equation E51, $\dfrac{I_F}{I_S} = \dfrac{R_{NP}(F)}{R_{NP}(S)} \approx 1 + \Delta e \dfrac{d}{de} \ln R_{NP}.$

Since the total extinction is only possible with a perfect antiglare support and since this support only exists for a normal incidence, the performances of this support for convergent light visualisation, i.e. for imaging purposes, are limited.

VII.2) Between crossed polariser and analyser

The equation E51 becomes:

$$\frac{R_F(\phi)}{R_S(\phi)} \approx 1 + \Delta e \frac{d}{de} \ln |\sigma|^2 \qquad \text{(E52)}$$

An optimal contrast is therefore obtained for both situations:

i) $\dfrac{d}{de} \ln |\sigma|^2 \rightarrow +\infty$ (extinction of the support, $C_f = +1$)

ii) $\dfrac{d}{de}\ln |\sigma|^2 = -1$ (extinction of the film, $C_f = -1$)

Each of these situations corresponds to $\sigma = 0$ i.e. at an ideal support AR-X-Pol. In the first equation, the optimal thickness e is that of the coating on its own. In the second equation, it is the sum of the thicknesses of the coating and of the object.

The sensitivity of the visualisation is given by, in [Angströms^{-1}]:

$$\frac{C_F}{\Delta e} = \frac{d}{de}\ln |r_p + r_s|$$

There results from the previous considerations that:

1) the best contrast amplifying supports are the ideal antiglare supports.

2) the supports AR-X-Pol are the single supports capable of providing total extinction in annular illumination and a quasi-perfect extinction with little convergent illumination. The use of these supports between crossed polarisers enables to obtain noticeably better contrasts than all other supports in all the imaging modes without labelling using incoherent illumination.

VIII. Non ideal contrast amplifying supports in polarised light:

VIII.1) Between crossed polarisers

When the support is not ideal any longer, the equation E19 bis is not satisfied any longer. It is in particular the case when the index of the layer 2 is imposed and it has no solution any longer because the relation E26 is not verified any longer. Then, the total extinction is not possible any longer and the condition $C_f = +1$ may not be reached any longer (the condition $C_f = -1$ may still be possible for a very particular object). We show that the contrast is not optimised any longer when the minimum of the reflected intensity is reached, but for thicknesses of layer 2 situated on both sides of this minimum, the minimum corresponding to the sign inversion of the contrast.

In this case, the contrast is optimised when:

i) $\frac{d}{de}\ln|\sigma|^2$ maximum (partial extinction of the support, C_f maximum)

ii) $\frac{d}{de}\ln|\sigma|^2$ minimum (partial extinction of the film, C_f minimum)

I.e. when :

$$\frac{d^2}{de^2}\ln|\sigma|^2 = 0 \quad (E57)$$

We shall limit ourselves to the case of dielectric materials and we are looking for the optimal thickness of the dielectric layer 2 for optimising the contrast with an amplifying coating made of a single layer 2.

The thickness of contrast inversion e_1 is given by the equation E24 which corresponds to the condition that $e^{-2j\beta_1}$ is real. In order to explore the thicknesses situated on both sides of e_1, we assume $\beta_1 = \frac{\pi}{2} + \varepsilon$.

As the relation E26 is not completely verified any longer, we assume:

$$\Delta \equiv \frac{\sigma_{12}(1+\pi_{01}) - \sigma_{01}(1+\pi_{12})}{\sigma_{01}} \quad (E58)$$

The development in ϵ of $|\sigma|^2$ gives:

$$|\sigma|^2 = \sigma_{01}^2 \left(\frac{A+4B\varepsilon^2}{C+4D\varepsilon^2}\right) \quad (E59)$$

an expression wherein:

$$A = \Delta^2 \quad (E60)$$

$$B = (1-\pi_{12})^2 + \Delta(1+\pi_{12}) \quad C = (1 - P + \pi_{01}\pi_{12}) \quad D = P + P\pi_{01}\pi_{12} - 4\pi_{01}\pi_{12}$$

and where :

$$P = r_{01(p)}r_{12(p)} + r_{01(s)}r_{12(s)} \quad (E61)$$

There follows:

$$\frac{d}{d\varepsilon}\ln|\sigma^2| = \frac{8B\varepsilon}{A+B\varepsilon^2} - \frac{8D\varepsilon}{C+D\varepsilon^2} \quad (E61)$$

The solution of the equation E57 is:

$$4\varepsilon^2 = \frac{AC}{BC + AD} \qquad \text{(E62)}$$

If the coating remains ▮▮ ▮▮ ▮▮▮▮▮, Δ is small, and in such a case ϵ is close to $\frac{\Delta}{2}$.

5   The relation of definition E18 provides finally with both thicknesses e_1' and e_1'' which optimise the contrast:

$$e_1' = \frac{\lambda}{4}\left(1 + \frac{2\varepsilon}{\pi}\right) + k\frac{\lambda}{2} \qquad \text{(E63)}$$

10

$$e_1'' = \frac{\lambda}{4}\left(1 - \frac{2\varepsilon}{\pi}\right) + k\frac{\lambda}{2}$$

The formulas E63 generalise the formula E24 to the non ideal contrast amplifying supports.

We shall designate the family of the contrast amplifying supports by the
15   designation supports Ampli-Pol when it is advisable to differentiate it from the supports AR-Pol. Between crossed polarisers, they become the supports Ampli-X-Pol, etc..

IX. Anisotropic supports AR-X-Pol and Ampli-X-Pol

20   In any use of a support AR-Pol or of a support Ampli-Pol, it is essential to optimise its performances (useful extinction rate C_u or contrast C_f). To do so, one should comply with the relations which define it with precision, which generates difficulties of manufacture (strict tolerance on the thickness e for example) and difficulties of implementation (adjustment of the parameters of
25   the microscope for visualisation for instance).

In order to remedy these difficulties, it is advantageous to introduce in the supports an adjustment element. This element is provided by the use of materials with anisotropic optical properties in the realisation of the supports.

By default, the anisotropic material is the substrate 1, the layer 2 and the
30   incident medium remaining isotropic. The main axes (x, y, and z) of the anisotropic material are parallel and perpendicular to the surface of the

support. The Fresnel coefficients of the support are replaced with a reflection matrix:

$$\begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix} \qquad (E70)$$

5

wherein, taking into account, the orientations of the main axes $r_{ps} = r_{sp} = 0$, which enables to express the reflected amplitude relative to the incident amplitude as:

10
$$\begin{pmatrix} E_{r_p} \\ E_{r_s} \end{pmatrix} = \begin{pmatrix} r_{px} & 0 \\ 0 & r_{sx} \end{pmatrix} \begin{pmatrix} E_{ip} \\ E_{is} \end{pmatrix} \qquad (E71)$$

when the axis x is in the plane of incidence, and as:

$$\begin{pmatrix} E_{r_p} \\ E_{r_s} \end{pmatrix} = \begin{pmatrix} r_{py} & 0 \\ 0 & r_{sy} \end{pmatrix} \begin{pmatrix} E_{ip} \\ E_{is} \end{pmatrix} \qquad (E71)$$

15

when the axe y is in the plane of incidence. At the interface between the isotropic medium i and the anisotropic medium j, the coefficients $r_{ij(px)}$, $r_{ij(sx)}$, $r_{ij(py)}$ and $r_{ij(sy)}$ are given by

20
$$r_{ij(px)} = \frac{n_{jx} n_{jz} c_i - n_i \left(n_{jz}^2 - n_i^2 s_i^2\right)^{\frac{1}{2}}}{n_{jx} n_{jz} c_i + n_i \left(n_{jz}^2 - n_i^2 s_i^2\right)^{\frac{1}{2}}} \qquad (E72)$$

$$r_{ij(sx)} = \frac{n_i c_i - \left(n_{jy}^2 - n_i^2 s_i^2\right)^{\frac{1}{2}}}{n_i c_i + \left(n_{jy}^2 - n_i^2 s_i^2\right)^{\frac{1}{2}}}$$

if the plane of incidence is parallel to x, and:

$$r_{ij(py)} = \frac{n_{jy} n_{jz} c_i - n_i \left(n_{jz}^2 - n_i^2 s_i^2\right)^{\frac{1}{2}}}{n_{jy} n_{jz} c_i + n_i \left(n_{jz}^2 - n_i^2 s_i^2\right)^{\frac{1}{2}}} \qquad (E73)$$

$$r_{ij(sy)} = \frac{n_i c_i - \left(n_{jx}^2 - n_i^2 s_i^2\right)^{\frac{1}{2}}}{n_i c_i + \left(n_{jx}^2 - n_i^2 s_i^2\right)^{\frac{1}{2}}}$$

if the plane of incidence is parallel to y.

The equations E72 show that, according to the relative orientation $\Phi = (P,x)$ of the sample and of the polariser P in azimuth, obtaining by rotating one element with respect to the other around the normal, one obtains variable effective Fresnel coefficients $r_{ij(p)}$ and $r_{ij(s)}$, given by the following linear combinations:

$$r_{ij(p)} = r_{ij(px)} \cos\varphi + r_{ij(py)} \sin\varphi \qquad (E74)$$

$$r_{ij(s)} = r_{ij(sx)} \cos\varphi + r_{ij(sy)} \sin\varphi$$

An analogue reasoning applies to the case where the substrate 1 is isotropic and the layer 2 anisotropic.

Thus, a support optimal for visualisation under microscope in polarised light is anisotropic and such that the optimal value of its reflection coefficients r_p and r_s is obtained for an intermediate angle Φ between 0 and $\frac{\pi}{2}$ and preferably equal to $\frac{\pi}{4}$ to provide maximum freedom of adjustment.

X . Circular polarisation

The invention may not be limited to the foregoing description. The light beam used has thus, in another embodiment a circular polarisation and the supports AR-Pol and Ampli-Pol exhibit advantageously the same efficiency. As it is well known, the observations and measurements between crossed polariser and analyser are then replaced with equivalent observations and measurements between [polariser followed by a $\frac{1}{4}$ wave plate] and [identical $\frac{1}{4}$ wave plate followed by a polariser (or analyser) parallel to the former, preferably identical]. This is also valid for all the techniques of visualisation and of measurement using differential interferential contrast (DIC).

The support of the invention has been subject to several implementations presented in the following examples for which the random illumination wavelength is, under otherwise specified, λ = 540 nm.

It has been observed that when a support complying with the invention has been determined, it is possible to deduce therefrom other supports which, from the invention viewpoint, have comparable properties by modifying the thickness e_1 of the layer 2 and the length λ of use while maintaining their ratio $\frac{e_1}{\lambda}$ constant. This opens very wide possibilities.

Example 1

The supports AR-Pol, although obeying very stringent construction rules, can be varied endlessly. Four examples (index n_l and thickness of layer e_l(Å)) of supports AR-Pol-1 ideal for an incidence close to the normal (θ_0=5°) are presented below. These examples are intended for use as object-carrying supports for observations and measurements conducted in a surrounding medium such as air (the incident medium being the air or the substrate).

Substrate	n_2	n_1	e_1(Å)
Gold	$0.40 - 2.6j$	1.70	694
Silver	$0.13 - 3.44j$	1.59	795
Aluminium	$0.92 - 0.95j$	2.01	346
Nickel	$1.76 - 3.2j$	1.51	847

We also give for these substrates 1 the indices n_1 and the thicknesses e_1 of the single layer 2 coatings which optimise the contrast C_f of any observation between crossed polarisers with a convergent axial illumination under an incidence of 0,2° (by convention θ_0 = 0) and we give the value n_1 and the absolute value of the contrast C_f obtained with convergent illumination with 30° opening at the edge of a film with 1 nanometre thickness as well as the thicknesses e_1' and e_1" as defined by the equations 63.

The index n_2 is the index of the material forming the support. It is extracted from « Hand Book of Optics, Mc Graw Hill Professional Publishing New York 2000 » wherein a particular wavelength is defined for each material.

		$\theta_0=0$	$\theta_0=0$		$0<\theta_0<\frac{\pi}{6}$	$0<\theta_0<\frac{\pi}{6}$	$0<\theta_0<\frac{\pi}{6}$	
Substrate	n_2	n_1	e_1(Å)	C_f	n_1	e_1'(Å)	e_1''(Å)	C_f
Gold	0.47 − 2.83j	1.65	800	1	1.58	870	880	0.17
Silver	0.2 − 3.44j	1.60	850	1	1.53	930	940	0.17
Aluminium	1.44 + 5.23j	1.50	953	1	1.44	1 040	1 050	0.07
Nickel	1.58 − 3.42j	1.52	920	1	1.46	1 000	1 010	0.06

Besides, we also give, for these substrates 1, the indices n_1, thicknesses e_1 of the single layer 2 coatings which optimise the contrast C_f of any observation between crossed polarisers with annular illumination under an incidence of 20°. We give the absolute value of the contrast C_f obtained under these conditions, with convergent illumination with 30° opening ($\pi/6$) at the edge of a film with 1 nanometre thickness. The thicknesses e_1' and e_1'' are those given by the equations E63.

		$\theta_0=20°$	$\theta_0=20°$		$0<\theta_0<\frac{\pi}{6}$	$0<\theta_0<\frac{\pi}{6}$	$0<\theta_0<\frac{\pi}{6}$	
Substrate	n_2	n_1	e_1(Å)	C_f	n_1	e_1'(Å)	e_1''(Å)	C_f
Gold	0.40 − 2.6j	1.64	739	1	1.64	730	740	0.65
Silver	0.13 − 3.44j	1.55	838	1	1.55	832	842	0.85
Aluminium	0.92 − 0.95j	1.89	399	1	1.89	395	415	0.25
Nickel	1.76 − 3.2j	1.48	890	1	1.48	880	905	0.50

The indices n_1 are very common and the layers 2 may be realised by all the conventional deposition techniques, for example oxide depositions realised by PECVD (Phase evaporation chemical vapour deposition).

Example 2

Examples (index n_1) of supports AR-Pol-1 are presented below for a substrate 1 of silicium and for uses either in a surrounding medium such as air, or in immersion.

Ideal anisotropic supports AR-Pol may be obtained by depositing on a
5   substrate 1 of silicium ($n_2 = 4.12 - 0.05j$) doped or not, cleaved according to a plane 100, a layer 2 of optical thickness $\frac{\lambda}{4}$ according to the relation E24 and of index n_1 equal to:

1.37 when the incident medium is air (index 1)

1.79 when the incident medium is water (index 1.33)
10   1.99 when the incident medium is oil (index 1.5).

The layers 2 then have less common indices which are obtained preferably:

- either by sol-gel and aerogel process leading to porous silicas (index 1.37), the index also being approximated correctly by a layer of MgF_2
15   (index 1.38).

- or by oxidation techniques with gas mixtures leading to material mixtures $SiO_2 - SiO$ (index 1.79), or to oxinitride layers SiO_xN_y (indices 1.79 and 1.99) with the following proportions: x= 0.4 and y= 0.6 to obtain $n_1 = 1.79$; x=0 and y= 1 to obtain $n_1 = 1.99$.

20   The index $n_1 = 1.37$ may also be obtained by depositing fluorinated polymers (such as trifluoroalkyl-alkylsiloxanes or trifluoroalkyl-alkylsiloxane and dimethylsiloxane copolymers), whereas the deposition may be conducted by spin-coating from a solution.

These indices 1.79 and 1.99 may also be obtained by all the deposition
25   techniques, in particular by PECVD deposition of oxide mixtures such as HfO_2 and Y_2O_3 or of all the materials listed in the dictionaries of physical properties of the materials (for example: Handbook of Optical constants of solids, Vol. 1-5, Academic Press, Ed. Palik and Ghosh (1997)) or in the optics dictionaries (for example: Handbook of optics, McGraw-Hill Professional Publishing, New-
30   York, (2000)).

The index of certain materials changes significantly with the wavelength of the light, and the adjustment of index may be replaced with an adjustment of wavelength; it is for example the case of a layer 2 of SiO which exhibits a useful index of 1.95 for λ = 490 nm and of 1.99 for λ = 540 nm.

35

Example 3

We give below a few other examples of ideal supports AR-Pol (which are therefore also ideal Ampli-Pol) for different incident media and for an annular illumination of very small incidence (θ_0 = 5 degrees). The thickness e_1 is in Angströms.

A first table is given for an angle of incidence of 0,2°. The index n_2 is the index of the material forming the support. It is extracted from « Hand Book of Optics, Mc Graw Hill Professional Publishing New York 2000 wherein a particular wavelength is defined for each material.

Substrate	n_2	n_0	n_1	e_1
Gold	0.47-2.83j	1.33	2.42	490
Gold		1.5	2.95	368
Silver	2.0-3.44j	1.33	2.35	526
Silver		1.5	2.8	417
Aluminium	1.44+5.23j	1.33	2.05	684
Aluminium		1.5	2.33	587
Nickel	1.58-3.42j	1.33	2.13	620
Nickel		1.5	2.5	505
Cadmium	1.13-5.01j	1	1.47	968
Cadmium		1.33	2.08	667
Cadmium		1.5	2.38	575
Tin	1.43-5.25j	1	1.48	975
Tin	1.48-5.25j	1	1.48	975
		1.33	2.03	693
		1.5	2.34	600
copper	0.52-2.57j	1	1.68	768
copper		1.33	2.52	448
copper		1.5	3.05	338
Iron (evaporated)	1.51-1.63j	1	1.545	802
		1.33	2.24	459
		1.5	2.72	332

A second table is given for an angle of incidence of 5°.

The wavelength is λ = 540 nm except for the Cadmium where it is equal to 589.3 nm.

Substrate	n_2	n_0	n_1	e_1
Gold	$0.40 - 2.6j$	1.33	2.42	490
Gold	$0.40 - 2.6j$	1.5	1.79	755
Silver	$0.13 - 3.44j$	1.33	2.28	512
Silver	$0.13 - 3.44j$	1.5	2.7	412
Aluminium	$0.92 - 0.95j$	1	1.89	399
Nickel	$1.76 - 3.2j$	1.33	2.11	572
Nickel	$1.76 - 3.2j$	1.5	2.45	473
Cadmium	1.13-5.01j	1	1.49	970
Cadmium	1.13-5.01j	1.33	2.05	684
Cadmium	1.13-5.01j	1.5	2.36	582
Tin	1.48-5.25j	1	1.48	899
Tin	1.48-5.25j	1.33	2.02	640
Tin	1.48-5.25j	1.5	2.33	548
Copper	1.04-2.59j	1	1.62	746
Copper	1.04-2.59j	1.33	2.23	423
Copper	1.04-2.59j	1.5	2.83	351
Iron (evaporated)	1.51-1.63j	1	1.54	737
Iron (evaporated)	1.51-1.63j	1.33	2.23	423
Iron (evaporated)	1.51-1.63j	1.5	2.72	305

5

Example 4

Finally, we give an example with an annular illumination at a wavelength λ = 589.3 nm and a significant angle of incidence. For a support of cadmium
10 and observation in a surrounding medium such as air under a single angle of incidence of 30 degrees, the ideal layer 2 is obtained for n_1 = 1.42 and e_1= 1084 Angströms.

The values of the indices are deduced from the book of Born and Wolf, entitled "Principles of Optics : Electromagnetic Theory of Propagation,
15 Interference and diffraction of Light", Cambridge University Press (1999) and

from the book of E.D. Palik, entitled « Handbook of optical constants of solids », vol. 1 to 5, Academic Press (1985).

Applications are possible in the industries of the optics, for improving measurements or the reflection-based observation of thin films or very small
5   objects under optical microscope or under any other optical imaging instrument such as visor, spectacles, macroscope, magnifying glass, binocular magnifying glass, camera, photo camera, near field microscope, endoscope, optical near field microscope, biochip reader, magneto-optical reader, confocal microscope.

10   The supports AR–X-Pol and Ampli-Pol are usable simultaneously as antiglare background and as object-carrying support for all microscopy operation in reflection polarised light, regardless whether the observation is conducted in open air, in immersion, or through the support.

They enable optimal visualisation not only of objects placed on the
15   surface of the support, but also of the interface between the substrate 1 and the layer 2.

They enable optimal visualisation and measurement of all the effects of dichroism and of birefringence inside the layer 2 properly speaking.

This is particularly advantageous for parallel imaging and reading of
20   magnetic fields situated in the layer when it possesses magneto-optic susceptibility at each of the following effects: Faraday effect, Voigt effect, linear magnetic birefringence.

They enable optimal visualisation and measurement of all the reflection anisotropic effects at the interface between the substrate 1 and the layer 2,
25   which is particularly advantageous for implementing the anisotropic reflection microscopy technique (RAM).

This is also particularly advantageous if the substrate 1 exhibits magneto-optic susceptibility (longitudinal or transverse Kerr effect, polar Kerr effect), the coatings AR-Pol providing considerable improvement in contrast
30   and in sensitivity of the magneto-optic reading process.

The supports AR-Pol are particularly efficient for reflection differential interferential contrast observations, regardless of the variation of the technique used, and regardless of the type of polarisation used (linear or circular) and enable the combination of all these interferential contrast

techniques with all the other visualisation, detection or measurement techniques mentioned above.

The single layer 2 coatings AR-Pol exist for all the types of substrates 1. They enable more sensitive detection than with all the other modification techniques which take place at the end of an optic fibre, and notably the detection of a species captured by a sensitive layer 2 installed at the end of such a fibre.

These are ideal object-carrying supports for near field microscopy techniques (AFM, STM, SNOM, and other SPM) since they enable detection, localisation and visualisation of films or objects invisibles by conventional optical techniques.

These are also ideal supports to conduct in situ optical tracking or to control a posteriori the depositions of ultra thin layers 2 performed by all the deposition techniques available, for example by the Langmuir-Blodgett technique, by plasma, ionic deposition, spin-coating, dip-coating, MBE techniques, etc...

Used as object-carrying supports, they also increase the efficiency of magneto-optic techniques, polarised light cofocal microscopy techniques, SNOM techniques (Scanning Near-field Optical Microscopy) in polarised light, and of all the spectroscopic visualisation techniques (Infrared Absorption, Raman, Fluorescence, ultra-violet Absorption, 2-photon Microscopy).

Used as object-carrying supports, they enable to increase considerably the quality of reflectivity measurements in polarised light and of ellipsometry under optical microscope.

The supports AR-Pol are also advantageously used as object-carrying supports in all the micro-manipulation devices under optical microscope: optical tweezers, magnetic tweezers, piezoelectric tweezers.